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A Study on Combination of Two Friction Dampers to Control Stayed-cable Vibration under considering its Bending Stiffness

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Abstract. The stayed-cable is one of vital component of cable-stayed bridges. Stayed-cable is often very long with a small diameter and low mass, which can be considered horizontal flexible structure with very low natural frequency. Under the influence of cyclic load in specific conditions, stayed-cable can store the energy and vibrate with large amplitude. This paper focuses on studying the methods of two-friction damper combination for reducing the cable vibration, and evaluates the capacity of friction-damper parameters in mitigating the vibration of stayed-cable. The results show the relationship between the damping factor of stayed-cable and various parameters such as Equivalent viscous constants, friction, spring constant, points attached damper to stayed-cable. For long-span cable-stayed bridges, cable has relatively large diameter and it is normally covered by grouting mortar or Epoxy. Consequently, its bending stiffness is considerably increased. Therefore, it is necessary to take into account its bending stiffness during the vibration analysis process. From these results, designers can assess and choose the attaching point as well as parameters of friction damper, which are optimal for specific stayed-cable.

Keywords: Stayed-cable, damping ratio, natural vibration frequency, friction damper.

1 Introduction

Stayed-cable is one of vital component of cable-stayed bridges. During its lifetime, stayed-cable usually vibrates strongly under some cycle loads such as: wind load, live loads, interaction of rain and wind loads… then it will be destroyed by fatigue phenomenon. To mitigate the vibration of cables, some damper devices are used as a damping countermeasure: Viscous damper (VD) [1], [2], [3]; High damping rubber damper (HDRD) [4], Friction damper (FD) [5]. However, due to aesthetical reasons, the damper distance is constrained within a few percent of the cable's length, and the damper performance is then quite limited. As a result, in application to long-span cable-stayed bridges, such as Tatara (Japan) or Stonecutters (Hong Kong), where the longest cables are over 500m, it may be tough to attain a desired level of modal damp-

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ing with a single damper allocation. To increase the modal damping, using two units of dampers at different locations may constitute a possible solution. In this paper, authors focus on how to reduce the vibration of stayed-cable using combination of two friction dampers. In addition, the Matlab program was also generated to design damper devices. The Matlab program can also optimize parameters of damping device and the suitable location at which the damping devices should be attached. The taut cables in long-span Cable-stayed bridges which have relatively big diameter, is especially covered by mortar or epoxy and thus cable' bending stiffness increases significantly. Therefore, it is necessary to take into account the bending stiffness of cable during the analysis of its vibration.

2 Model of a Stayed-cable attached two friction dampers

Given a cable with tension force (S), length (L) and the mass of damper is G_i. Two friction dampers are installed at both ends of cable. Each friction of damper is modeled by a viscous force connecting with a spring force as Fig. 1

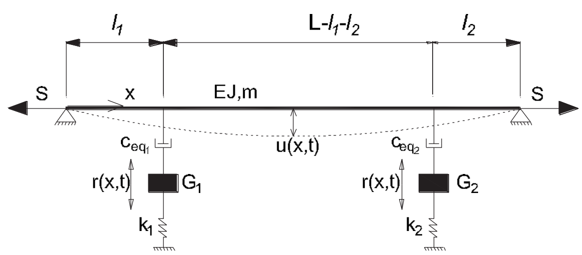

Fig. 1. A Schematic illustration of a stayed-cable attached to two Friction Dampers

It is assumed that the cable tension force is much larger than its weight and the friction between cable and the air is negligible. Taking in to account the bending stiffness of cable and the mass of damper, as Duy-Thao at el [6] and VSL [7] the differential equation of free vibration of the system cable with damper devices are:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} + a^2 \dfrac{\partial^4 u}{\partial x^4} - 2s^2 \dfrac{\partial^2 u}{\partial x^2} = c_{eqi} \cdot \dfrac{\partial (u-r)}{\partial t}(x,t)\delta_{Xdi}(x) & (1) \\ G_i \dfrac{d^2 r}{dt^2} + k_i r(t) - c_{eqi} \dfrac{d(u(X_{di}) - r)}{dt}(t) = 0 & (2) \end{cases}$$

Where $u=u(x,t)$-the displacement of cable at point x; δ-the Dirac's function; $r(x,t)$-the displacement of damper device; X_d-the point attached damper device; G_i-the mass of i^{th} damper device; k_i and c_{eqi} are the spring constant and the equivalent viscous constant of i^{th} damper device.

Use the separation of variables with Fourier series, the solution of Eq. (1) can be found as:

$$u(x,t) = X(x)T(t) \tag{3}$$

From Eq. (2), the differential equation of damper device is rewritten as Eq. (4):

$$G_i \ddot{r}(t) + c_{eqi}.\dot{r}(t) + k_i r(t) = X(l_i).c_{eqi}.\mathrm{i}.\eta.e^{i\eta t} \tag{4}$$

The solution of Eq. (4) has the form as follow

$$r(t) = \frac{X(l_i).2\delta_i.\mathrm{i}.\eta}{(\omega_i^2 + 2.\delta_i.i.\eta - \eta^2)}.e^{i\eta t} \tag{5}$$

From the boundary conditions and the equilibrium in the vertical direction at the points viscous damping device attached to, we can get the frequency equation of system cable and damper device:

$$\begin{aligned}
&\left(\frac{EJ.m}{S^2}\eta^2 - 1\right)^2 \left[\cot(\eta\sqrt{\frac{m}{S}}l_1).\cot[\eta\sqrt{\frac{m}{S}}(L-l_1-l_2)] + \cot(\eta\sqrt{\frac{m}{S}}l_1).\cot(\eta\sqrt{\frac{m}{S}}l_2) + \cot(\eta\sqrt{\frac{m}{S}}l_2).\cot[\eta\sqrt{\frac{m}{S}}(L-l_1-l_2)] - 1\right] - \\
&\left(\frac{EJ.m}{S^2}\eta^2 - 1\right)\left[\begin{array}{l}[\cot[\eta\sqrt{\frac{m}{S}}(L-l_1-l_2)] + \cot(\eta\sqrt{\frac{m}{S}}l_2)]\frac{c_{eq1}}{\sqrt{mS}}\left(i + \frac{2\delta_1.\eta}{(\omega_1^2 + 2.\delta_1.i.\eta - \eta^2)}\right) + [\cot[\eta\sqrt{\frac{m}{S}}(L-l_1-l_2)] + \\ + \cot(\eta\sqrt{\frac{m}{S}}l_2)]\frac{c_{eq2}}{\sqrt{mS}}\left(i + \frac{2\delta_2.\eta}{(\omega_2^2 + 2.\delta_2.i.\eta - \eta^2)}\right)\end{array}\right] = \\
&-\frac{c_{eq1}}{\sqrt{mS}}\left(i + \frac{2\delta_1.\eta}{(\omega_1^2 + 2.\delta_1.i.\eta - \eta^2)}\right)\frac{c_{eq2}}{\sqrt{mS}}\left(i + \frac{2\delta_2.\eta}{(\omega_2^2 + 2.\delta_2.i.\eta - \eta^2)}\right)
\end{aligned} \tag{6}$$

where η_i- the frequency of cable at i[th] mode.
Given k as eigenvalue of cable with complex form, k is defined as:

$$k_i = \sigma_i + j.\varphi_i = \left(-\xi_i + j.\sqrt{1-\xi_i^2}\right)\eta_i / \omega_1^0 \; ; \xi_i = \sqrt{1 + \varphi_i^2 / \sigma_i^2} \tag{7}$$

where $\omega^0{}_1$- the natural frequency of cable without damping device; ξ_i- the damping ratio of cable at i[th] mode.
Substitute Eq. (7) into Eq. (6) and separating into real and imaginary part, we can get the system of equation (8) and (9). Where $A = \Delta.\left(\sigma^2 - \phi^2\right)$; $B = \Delta.\sigma.\phi$; $a = \pi.\left(1 - l_1/L - l_2/L\right).\sigma$; $b = \pi.\left(1 - l_1/L - l_2/L\right).\phi$; $c = \pi.\sigma.l_2/L$; $d = \pi.\phi.l_2/L$; $e = \pi.\sigma.l_1/L$; $f = \pi.\phi.l_1/L$; $\Delta = \pi^2.EJ/(S.L^2)$; $\Theta_i = c_{eqi}/\sqrt{m.S}$; $\lambda = \omega^0{}_1$; $c_{eqi} = 4.F/(\pi.\omega.A_i)$; $F \approx A_{th}.\pi.S/L$; $\omega^0{}_1 = \sqrt{S/m}.\pi/L$; $M_1 = \cosh^2 b - \cos^2 a$; $M_2 = \cosh^2 d - \cos^2 c$; $M_3 = \cosh^2 f - \cos^2 e$; $T_1 = \frac{(\phi^2+\sigma^2)^2\lambda^4 - 2\lambda^3\delta_1\phi(\phi^2+\sigma^2) - 2\lambda^2\omega_1^2(\sigma^2-\phi^2) + \omega_1^2(\omega_1^2 - 2\delta_1\phi\lambda)}{[\omega_1^2 - 2\delta_1\lambda\varphi - \lambda^2(\sigma^2-\phi^2)]^2 + (2\delta_1\lambda\sigma - \lambda^2 2\varphi\sigma)^2}$;

$$T_2 = \frac{2\delta_1\lambda\sigma(\omega_1^2 - \lambda^2(\phi^2+\sigma^2))}{[\omega_1^2 - 2\delta_1\lambda\varphi - \lambda^2(\sigma^2-\phi^2)]^2 + (2\delta_1\lambda\sigma - \lambda^2 2\varphi\sigma)^2}; \quad T_3 = \frac{(\phi^2+\sigma^2)^2\lambda^4 - 2\lambda^3\delta_2\phi(\phi^2+\sigma^2) - 2\lambda^2\omega_2^2(\sigma^2-\phi^2) + \omega_2^2(\omega_2^2 - 2\delta_2\phi\lambda)}{[\omega_2^2 - 2\delta_2\lambda\varphi - \lambda^2(\sigma^2-\phi^2)]^2 + (2\delta_2\lambda\sigma - \lambda^2 2\varphi\sigma)^2};$$

$$T_4 = \frac{2\delta_2\lambda\sigma(\omega_2^2 - \lambda^2(\phi^2+\sigma^2))}{[\omega_2^2 - 2\delta_2\lambda\varphi - \lambda^2(\sigma^2-\phi^2)]^2 + (2\delta_2\lambda\sigma - \lambda^2 2\varphi\sigma)^2}.$$

$$
\begin{aligned}
&\left[A^2-2A-4B^2+1\right]\left[\frac{\sin 2e.\sin 2a-\sinh 2f.\sinh 2b}{M_1.M_3}+\frac{\sin 2c.\sin 2a-\sinh 2d.\sinh 2b}{M_1.M_2}+\frac{\sin 2e.\sin 2c-\sinh 2f.\sinh 2d}{M_2.M_3}-4\right]\\
&+\left[4B(A-1)\right]\left[\frac{\sin 2e.\sinh 2b+\sin 2a.\sinh 2f}{M_1.M_3}+\frac{\sin 2e.\sinh 2d+\sin 2c.\sinh 2f}{M_2.M_3}+\frac{\sin 2c.\sinh 2b+\sin 2a.\sinh 2d}{M_1.M_2}\right]\\
&-4B\left[-\left(\frac{\sin 2a}{M_1}+\frac{\sin 2e}{M_3}\right)\Theta_2.T_3-\left(\frac{\sin 2a}{M_1}+\frac{\sin 2c}{M_2}\right)\Theta_1.T_1+\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2f}{M_3}\right)\Theta_2.T_4+\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2d}{M_2}\right)\Theta_1.T_2\right]\\
&-2(A-1)\left[\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2f}{M_3}\right)\Theta_2.T_3+\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2d}{M_2}\right)\Theta_1.T_1+\left(\frac{\sin 2a}{M_1}+\frac{\sin 2e}{M_3}\right)\Theta_2.T_4+\left(\frac{\sin 2a}{M_1}+\frac{\sin 2c}{M_2}\right)\Theta_1.T_2\right]\\
&+4\Theta_1\Theta_2(T_2T_4-T_1T_3)=0
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\left[4B(A-1)\right]\left[\frac{\sin 2e.\sin 2a-\sinh 2f.\sinh 2b}{M_1.M_3}+\frac{\sin 2c.\sin 2a-\sinh 2d.\sinh 2b}{M_1.M_2}+\frac{\sin 2e.\sin 2c-\sinh 2f.\sinh 2d}{M_2.M_3}-4\right]\\
&-\left[A^2-2A-4B^2+1\right]\left[\frac{\sin 2e.\sinh 2b+\sin 2a.\sinh 2f}{M_1.M_3}+\frac{\sin 2e.\sinh 2d+\sin 2c.\sinh 2f}{M_2.M_3}+\frac{\sin 2c.\sinh 2b+\sin 2a.\sinh 2d}{M_1.M_2}\right]\\
&-2(A-1)\left[\left(\frac{\sin 2a}{M_1}+\frac{\sin 2e}{M_3}\right)\Theta_2.T_3+\left(\frac{\sin 2a}{M_1}+\frac{\sin 2c}{M_2}\right)\Theta_1.T_1-\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2f}{M_3}\right)\Theta_2.T_4-\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2d}{M_2}\right)\Theta_1.T_2\right]\\
&-4B\left[\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2f}{M_3}\right)\Theta_2.T_3+\left(\frac{\sinh 2b}{M_1}+\frac{\sinh 2d}{M_2}\right)\Theta_1.T_1+\left(\frac{\sin 2a}{M_1}+\frac{\sin 2e}{M_3}\right)\Theta_2.T_4+\left(\frac{\sin 2a}{M_1}+\frac{\sin 2c}{M_2}\right)\Theta_1.T_2\right]\\
&+4\Theta_1\Theta_2(T_1T_4+T_2T_3)=0
\end{aligned}
\tag{9}
$$

The system of Eq. (8) and Eq. (9) are the nonlinear equation. The Matlab program Cable-BKDN is generated to solve the system by iteration method.

3 Numerical analysis results

In order to evaluate the effects of friction dampers on reducing vibration of cable, various parameters of damping device such as equivalent viscous constant c_{eq}, spring constant k, mass of damper device G and location attached to damper have been investigated. With each parameter of fiction damper changes, the damping ratio of cable is solved from the system of Eq. (8) and Eq. (9). The following sections present efficiency of friction damper in reducing the vibration of cables.

3.1 Influence of C_{eq} on mitigating the vibration of cable

The equivalent viscous constant of friction damper is characterized by the non-dimensional coefficient $\Theta_i = c_{eqi}/(m.S)^{0.5}$. The investigation of variation of the first damping ratio of cable "ξ_1" with parameters Θ_1 = [0÷100] corresponding to Θ_2 = 3,7,10,23,30 (or Θ_2 = [0÷100] corresponding to Θ_1 = 2,8,10,20,30) and points damping device attached to $l_1/L = l_2/L = 0.03$, are shown in Fig. 3 and Fig. 4. From the results in the Fig. 3 and Fig. 4, it shows that the first damping ratio "ξ_1" increases along with the raise of Θ_i. However, after reaching the maximum point, "ξ_1" slowly decreases when Θ_i increases. Therefore, it is possible to optimize the value "c_{eqi}" of friction damper and determine the maximum damping ratio of cable.

Furthermore, the optimal value Θ_1 doesn't change when Θ_2 changes. The same thing also happens with Θ_2 when Θ_1 changes. It means that two friction dampers work independently of each other and the total damping effect is the sum of the contribution from single friction dampers

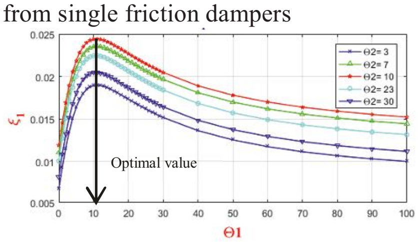

Fig. 2. Relationship between ξ_1-Θ_1

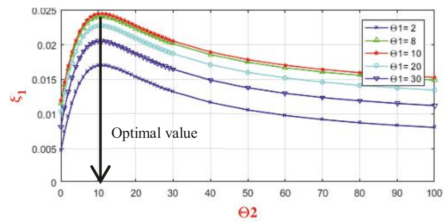

Fig.4. Relationship between ξ_1-Θ_2

3.2 Influence of the friction force on decreasing cable vibration

Friction of damper can have considerable effect on damping ratio of cable by value: $F=A_{th}.\pi.S/L$. The investigation of variation of the first damping ratio of cable "ξ_1" with parameters F = [0÷30] kN and vibration amplitude A=0.005, 0.01, 0.015, 0.02 (m), gives us the result showed in Fig. 5. From the results in the Fig. 5, corresponding to the vibration amplitude A and optimal friction F, the damping ratio of cable can reach the maximum value. When the friction exceeds the optimal friction (F) the damping ratio will decrease. The reason is that if friction is so high, the cable is locked at the point attached to damper. The clamping force between damper and cable will be very high; two components will not slide over each other and leading to not reduce vibration of cable.

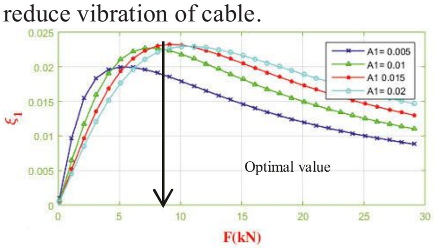

Fig.5. Relationship ξ_1-F_1 (various A_1)

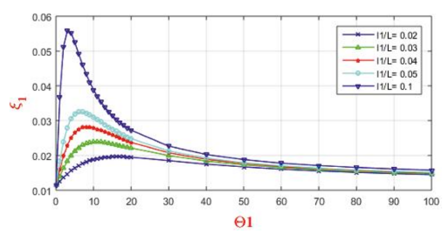

Fig.6. Relationship ξ_1- Θ_1 (various point attached damper l_1/L)

3.3 Influence of the attachment point on reducing vibration of cable

The investigation of variation of the first damping ratio of cable "ξ_1" with parameters Θ_1 = [0÷100] and the point attached damper l_1/L= 0.02, 0.03, 0.04, 0.05, 0.1, gives us the result showed in Fig. 6. It illustrates that the first damping factor "ξ_1" will be higher when the ratio l/L is larger. However, this value requires lifting up damping devices to a high position in comparison with the bridge deck. Therefore, this may bring about difficulties in designing, installing as well as maintaining damping devices and have influence on architectural bridges. Damping devices is usually installed at the point from 0.02 to 0.05.

3.4 Influence of the bending stiffness on reducing vibration of cable

The damping ratio of cable depends on cable characteristics as: bending stiffness EJ, tension force S and the length of cable L. The introduction of non-dimension parameter Δ is aimed at simplifying the investigations. The parameter Δ reflects the bending stiffness and be determined by $\Delta=\pi^2.EJ/(S.L^2)$. The investigation of variation of the first damping ratio "ξ_1" with parameters Δ = [0÷0.2] and points attached damper l_i/L=0.02; 0.03; 0.04, it shows that the first damping ratio gradually increases while Δ gradually increases. The rate of relative rise of the first damping ratio with l_i/L = 0.02 can reach more than 8%. This value is significant when considering the vibration of the cable.

4 Conclusion

This study focuses on application of two friction dampers in the aim of controlling the vibration of cable. The research also evaluated the influences of friction damper parameters in reducing the vibration of cable. The results showed the relationship between the damping ratio of cable and various parameters of damping device as: equivalent viscous constant c_{eq}; spring constant K, the relationship between the damping ratio and many different points damping device attached to, and the effect of bending stiffness of cable on mitigating its vibration by a non-dimension parameter Δ . Based on current study, structural engineer can assess and choose the attachment point as well as other parameters of two friction dampers, which are optimum for stayed-cable.

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